Reducing Cogging Torque in Surface-mounted Permanent Magnet Synchronous Motors by Non-uniform Distributed Magnets

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Abstract —In this paper, a novel method to reduce cogging torque in surface-mounted permanent magnet (PM) synchronous motors is proposed and studied. In this method all the PMs have the same dimensions, thus facilitate the mass manufacture. Firstly, the analytical expression of cogging torque when adopting this method was derived to analyze the detailed principle and influence of the proposed method on cogging torque. Then the optimal interval space between PMs was determined by the analytical method so as to reduce cogging torque. At last, in order to validate the effectiveness of the proposed method, Finite Element Method (FEM) was employed to calculate cogging torque of two prototype motors. The results proved that the proposed method can greatly reduce the cogging torque in PM motors.

I. INTRODUCTION

Permanent Magnet (PM) motors offer significant advantages in comparison with conventional motors. However, cogging torque, which is caused by the interaction between the magnetic poles and the stator teeth, is an inherent problem and results in mechanical resonance, vibration, and undesirable noise. Skewing is the most frequently used method to reduce cogging torque. However, it can not be used to design a motor which has short stack length. Many studies have been carried out to reduce cogging torque [1]-[6]. This paper proposed a novel method to reduce cogging torque by changing the interval space between two adjacent magnets, while all the dimensions of the PMs are the same, thus facilitate mass manufacture and minimize the cost. The investigation indicated that compared to the existing methods of magnet shifting and pole arc coefficient combination, the proposed method is more flexible for reducing cogging torque because it does not suffer from the constraint of the pole arc to pole pitch ratio and has less influence on the amplitude of back-EMF.

II. NON-UNIFORM DISTRIBUTED MAGNETS METHOD PROPOSED THIS PAPER

In PM motors, all the dimensions of PMs and the interval spaces between two adjacent PMs are normally all the same, as Fig.1 shows. This paper proposed a novel method for reducing cogging torque with non-uniform distributed magnets, as shown in Fig.2. In this method, the interval space between two adjacent magnets is different from that of the others, whereas all the magnets have the same dimensions, therefore, the method can be applied to a motor with any pole arc to pole pitch ratio.

As shown in Fig.2, the pole pairs of the motor is p, the width of PMs (PM_1-PM_6) is θ_c, the interval space width between PM_1 and PM_2 is θ_a, while that between the other $2p$-1 PMs is θ_b, thus

$$\theta_a + \theta_b(2p-1) + 2p\theta_c = 2\pi \tag{1}$$

If the ratio of θ_b to θ_a is defined as the interval space ratio K_t, θ_a can be expressed as

$$\theta_a = \frac{2\pi - 2p\theta_c}{(2p-1)K_t + 1} \tag{2}$$

It is obvious that, when K_t=1 is satisfied, all the interval space are of the same width, thus uniform rotor. Once the pole arc to pole pitch ratio is given, the interval space between two adjacent magnets can be determined by K_t.

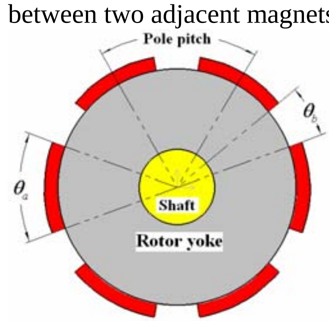


Fig. 1 Uniform distributed PMs

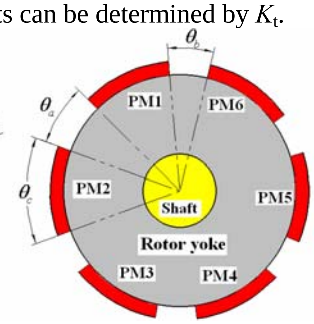


Fig. 2 Non-uniform distributed PMs proposed this paper

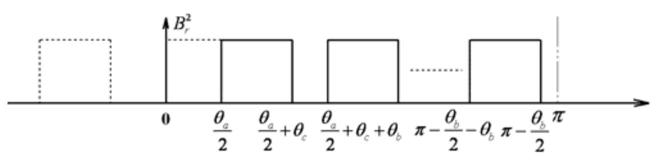


Fig. 3 Distribution of residual flux density when adopting the proposed method

III. STUDY OF THE INFLUENCE OF PROPOSED METHOD ON COGGING TORQUE BY ANALYTICAL METHOD

Cogging torque can be defined as the negative derivative of the magnetic co-energy W with respect to the rotor position angle α, i.e [1].

$$T_{cog} = -\frac{\partial W}{\partial \alpha} \tag{3}$$

Since the energy stored in iron can be negligible, the magnetic co-energy stored in the motor can be taken as that stored in the air gap and PMs, i.e [1].

$$\begin{aligned} W &\approx W_{gap} + W_{PM} = \frac{1}{2\mu_0}\int_V B^2 dV \\ &= \frac{1}{2\mu_0}\int_V B_r^2(\theta)\left(\frac{h_m(\theta)}{h_m(\theta)+g(\theta,a)}\right)^2 dV \\ &= \frac{1}{2\mu_0}\int_V B_r^2(\theta)G^2(\theta,\alpha)dV \end{aligned} \tag{4}$$

A. Fourier Expansion of $G^2(\theta,\alpha)$

When the slots distribute uniformly, the Fourier expansions of $G^2(\theta,\alpha)$ can be expressed in following forms

$$\left(\frac{h_m}{h_m+g(\theta,\alpha)}\right)^2 = G_0+\sum_{n=1}^{\infty} G_n\cos nz(\theta+\alpha) \tag{5}$$

Where z is the number of slots, G_n the n^{th} Fourier Transform coefficients of $\left(\frac{h_m}{h_m+g(\theta,\alpha)}\right)^2$.

B. Fourier Expansion of $B_r^2(\theta)$

When the method proposed above is adopted, the Fourier expansion of $B_r^2(\theta)$ should be expanded at the interval $[-\pi,\pi]$ as shown in Fig.3. According to (5) and the orthogonality of trigonometric function, only those of order the multiple of nz (n=1,2,3…) have effects on cogging torque. So the Fourier expansion coefficient B_{rn} can be derived as,

$$B_{rn}=\frac{2}{\pi}\sum_{i=1}^{p}\int_{\frac{\theta_a}{2}+(i-1)(\theta_b+\theta_c)}^{\frac{\theta_a}{2}+(i-1)(\theta_b+\theta_c)+\theta_c} B_r^2\cos n\theta d\theta$$

$$=\frac{2B_r^2}{nz\pi}2\sin\frac{nz\theta_c}{2}\frac{\sin\frac{nz}{2}p(\theta_b+\theta_c)}{\sin\frac{nz}{2}(\theta_b+\theta_c)} \tag{6}$$

$$\cos\frac{nz(\theta_a-2\theta_b-\theta_c)+nz(p+1)(\theta_b+\theta_c)}{2}$$

C. Analytical Model of Cogging Torque

Substituting (4)-(6) into (3), the analytical expression of cogging torque when adopting the proposed method in this paper can be derived as follows:

$$T_{cog}(\alpha)=\frac{\pi z L_{Fe}}{4\mu_0}(R_2^2-R_1^2)\sum_{n=1}^{\infty} nG_nB_{rn}\sin nz\alpha \tag{7}$$

In (7), the expression of B_{rn} is shown as (6), L_{Fe} is the stack length of armature, μ_0 the permeability of air, R_1 the outer radius of rotor yoke, R_2 the inner radius of stator.

D. Determining the Interval Space between Magnetic Poles by Analytical Method

According to the analysis above, B_{rn} can be calculated analytically with respect to different K_{t} according to (6). Fig.4 and Fig.5 show the calculated value of B_{rn} with respect to different K_{t} for 6-pole, 36-slot and 4-pole, 30-slot Permanent Magnet Synchronous Machines (PMSM). According to (7), since the cogging torque is proportional to the value of B_{rn}, if B_{rn} can be enable to be zero by adjusting the interval space between PMs, i.e. K_{t}, the cogging torque can be greatly reduced theoretically.

As shown in Fig.4 and Fig.5, either for the 6-pole, 36-slot or 4-pole, 30-slot prototype machine, there exists a value of K_{t} (K_{t} =0.619 for 4-pole 30-slot, K_{t} =0.423 for 6-pole 36-slot) which can enable all harmonic components of B_{rn} to be zero except those of orders the multiple of 2pnz. In other words, during a slot pitch, in addition to the components of orders 2pn (n=1,2,3…), all the harmonic components of cogging torque can be theoretically eliminated by the proposed method. For example, for the 4-pole, 30-slot prototype machine, the proposed method can theoretically eliminate all the harmonic components while the component of the order 4*n (n=1,2,3…) still exist. Similarly, for the 6-pole, 36-slot prototype machine, the component of the order 6*n (n=1,2,3…) exist.

In this case, since the higher the order of the component is, the less the amplitude is, the proposed method is more suitable to a machine with relatively large number of poles.

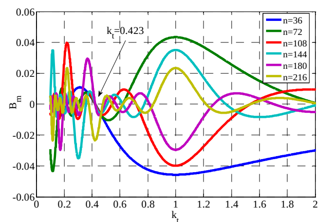


Fig. 4 B_{rn} calculated for 6-pole, 36-slot PMSM

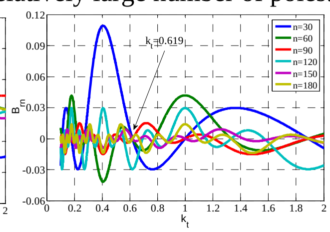
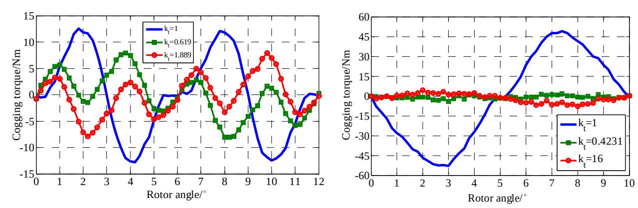


Fig. 5 B_{rn} calculated for 4-pole, 30-slot PMSM

IV. Validation by FEM

In order to validate the correctness and effectiveness of the analysis above, FEM is carried out to calculate the cogging torque of the PMSMs above corresponding to different K_{t}. The calculated results are shown in Fig.6. It can be seen that the cogging torque can be greatly reduced by the proposed method especially for the 6-pole, 36-slot machine which strongly verify the analysis and conclusions above.



(a) 4-pole, 30-slot PMSM (b) 6-pole, 36-slot PMSM

Fig. 6 Cogging torque calculated by FEM

V. References

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